Co-funded by the
Erasmus+ Programme
of the European Union

FRAME

Forest Resources Management and Planning

prof. Marušák, Dr. Kašpar, Dr. Panatiokidis, prof. Remeš

Co-funded by the
Erasmus+ Programme
of the European Union

FRAME

Forest Mensuration (Biometry)

It comes from the Latin word ‘***mensura***’ which means measure, in Greek the used term is ‘***dendrometria***’ comprises of two syllabes ‘***dendro***’ and ‘***metria***’ the first ones means ‘tree’ and the second one ‘metrics’

Co-funded by the Erasmus+ Programme of the European Union

FOREST MENSURATION

- One of the most fundamental disciplines within forest and related sciences
- Deals with the technical aspects of tree and forest stand measurements
  - measurement of tree variables - DBH, height
  - determination of form factor, age, basal area, tree volume
  - estimation of biomass, total and merchantable stand volume
- Deals with relations among tree/stand variables; instruments and tools
- Provides information at stand, local, regional and national level for forest management planning, forest policy decision, ...

Van Laar, A., Akça A., 2007: Forest Mensuration. Elsevier

Co-funded by the Erasmus+ Programme of the European Union

FRAME

Forest Mensuration

✓ We may say that **forest mensuration is the tool** which incorporates quantitative measurements of the forest to determine stand timber volume-productivity, health, and provides a basis off which management decisions can be made

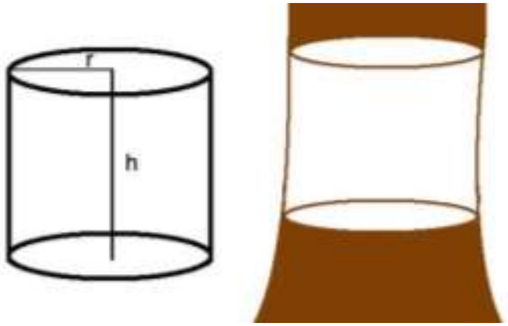
Scales of measurement-Methods

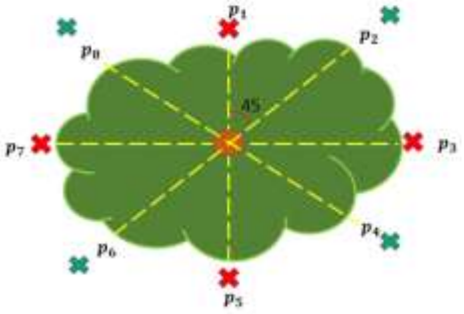
- Individual Tree level
- Forest stand level

Co-funded by the Erasmus+ Programme of the European Union

What to Measure & What Techniques Do Exist?

FRAME





Attributes	Methods	Techniques
Volume(s)	Direct	i.e., xylometer (bole)
	Indirect	Mathematical formulas Remote Sensing
Diameter(s)	Direct	Measuring Tape
	Indirect	Mathematical formulas Remote Sensing
Heights	Direct	✓ Vertical measuring pole ✓ Tape through climbing
	Indirect	✓ Trigonometric ✓ Geometric Remote Sensing
Crown Projection	Direct	✓ Measuring tape ✓ Laser devices & Ultrasonic etc.
	Indirect	Remote Sensing
Others: crown height; age; basal area; increment; growth etc.		

Co-funded by the Erasmus+ Programme of the European Union

Measurement Instruments

Analogue Caliper

Digital Caliper

Hypsometer

Relascope
Source: Source: https://en.wikipedia.org/wiki/Relascope

Measuring Tape

Laser Rangefinder

Suunto clinometer

Compass

Co-funded by the Erasmus+ Programme of the European Union

FRAME

Data Acquisition/Measuring Difficulties

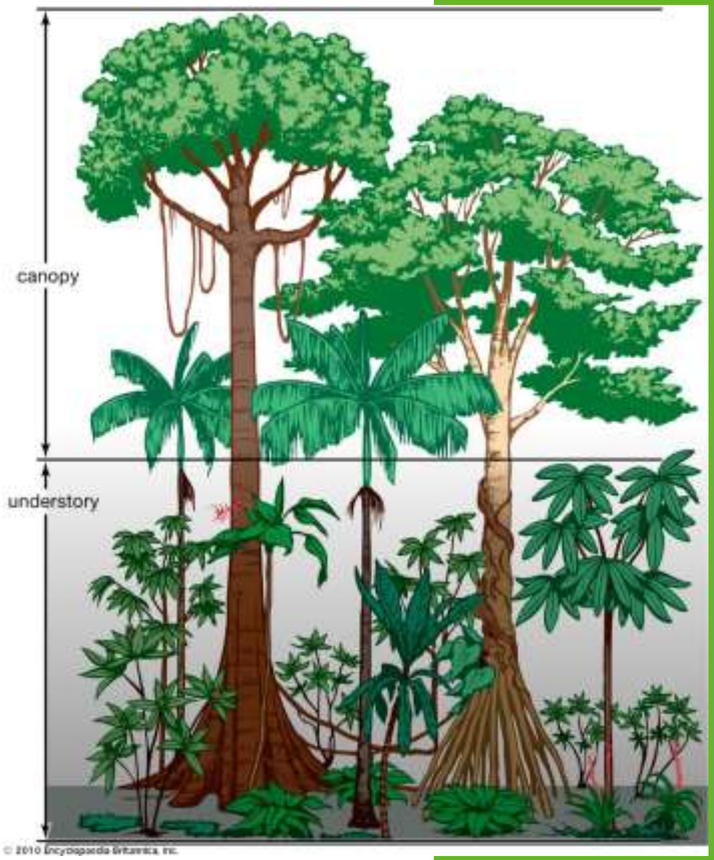
- In some cases measurements of trees/stands can be challenging but not impossible ☺

✓ High or dense ground vegetation i.e., shrub, grasses, etc.

✓ Biome – tree/forest stand structure
(i.e., estimation of height and diameter in tropical forests)

✓ Complicated forest structures - dense mixed forests types (crown overlapping especially in the determination of tree height and crown diameter) – stands with fallen trees etc.

✓ Site conditions i.e., topography



Source: https://www.britannica.com/science/tropical-rainforest/Population-and-community-development-and-structure

Co-funded by the Erasmus+ Programme of the European Union

Scales of Measurement

Different scales of measurement may be used for measuring tree and stand characteristics

- The nominal scale (nominal" scales could simply be called "labels) i.e. tree species, forest type, soil type etc.

- The next-strongest is ordinal scale i.e. soil type can be distinguished in poor, medium or good

- Metric scale (almost all forest mensuration characteristics, such as diameter, height, basal area, volume and increments, are continuous variables)

Co-funded by the Erasmus+ Programme of the European Union

Units of Measurement

Quantitative variables (measured in the metric scale)

- ✓ Data representing continuous variables should be recorded with an appropriate number of significant digits
- ✓ A tree diameter with a recorded diameter of 56 cm has two significant digits and implies that the tree has a diameter anywhere between 55.5 and 56.5 cm (~1 cm)
- ✓ When diameters are recorded in centimeters, no digits should be written to the right of the decimal point i.e., 43 cm BUT when recording in millimeters, there should not be more than one digit to the right of the decimal point i.e., 43.4 mm

- Diameter – cm or mm
- Length – m
- Area – sq.m (m^2)
- Volume – cu.m (m^3)
- Weight kg

Co-funded by the Erasmus+ Programme of the European Union

SI-Units Commonly Used in Forestry

a) Linear measures:

- Millimetre (mm)
- Centimetre (cm) 1 cm = 10 mm
- Decimetre (dm) 1 dm = 10 cm
- Meter (m) 1 m = 10 dm

b) Square measures:

- Hectare (ha) ---1ha = 10 000 m^2
- Square kilometre (sq km) ----1 km^2 = 100 ha

c) Cubic measures:

- Cubic meter (m^3) 1 m^3 = 1000 cubic decimetre (dm^2)

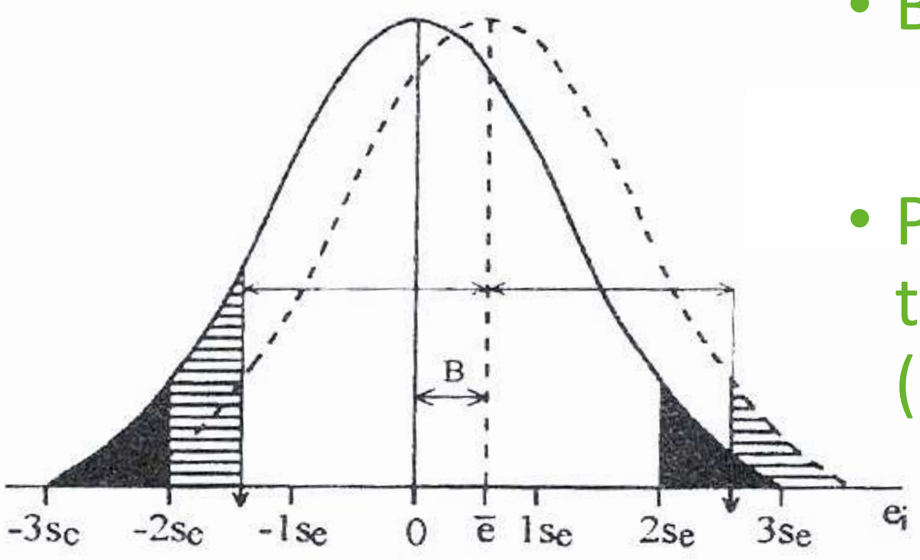
d) Weight measures:

- Kilogram (kg) 1 kg = 1000 g
- Tonne (t) 1 t = 1000 kg

Co-funded by the Erasmus+ Programme of the European Union

ESTIMATION

FRAME



- Bias

$$B = \bar{e} = \frac{\sum_{i=1}^{n} e_i}{n} d^2$$

- Precision – expresses closeness of the measurements to their mean (standard devitation)

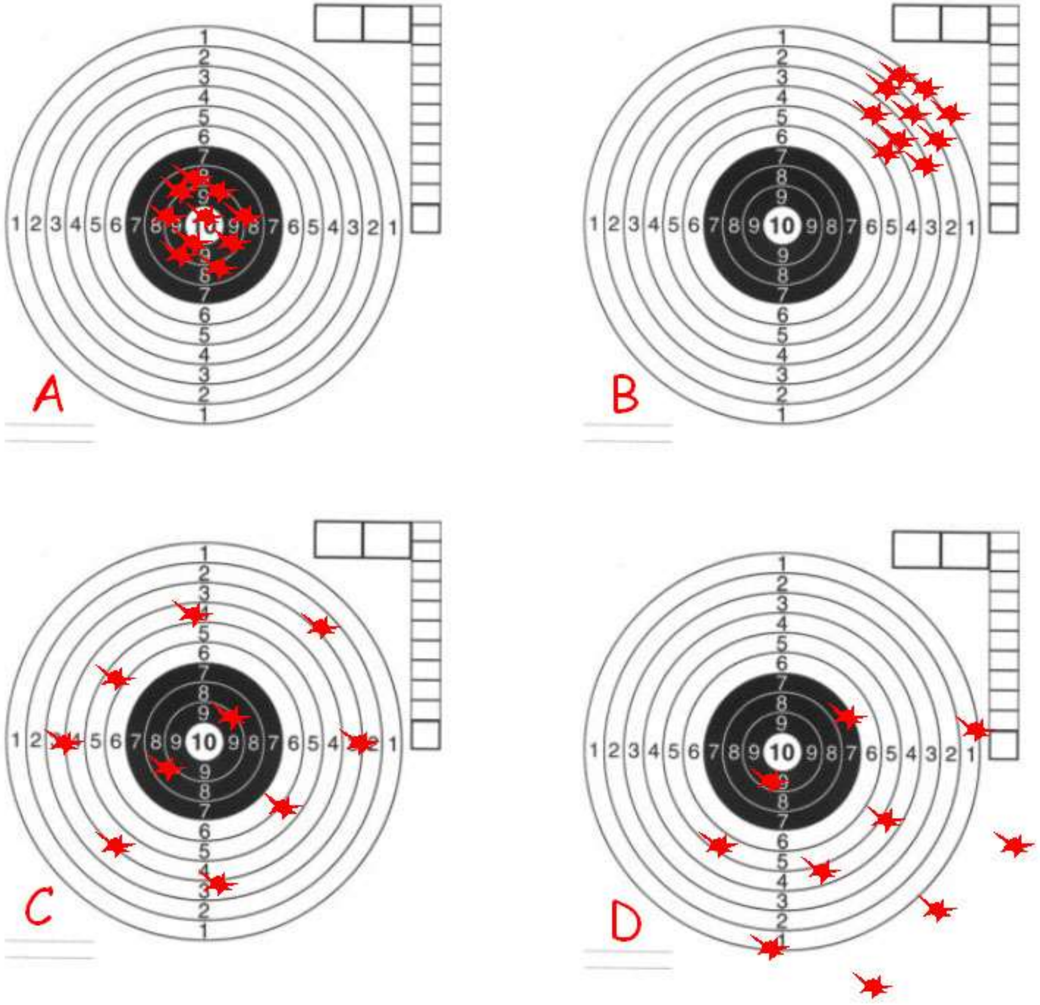
$$s_e = \sqrt{\frac{\sum_{i=1}^{n}(e_i - \bar{e})^2}{n-1}}$$

Accuracy – combines bias and prediction and expresses the closeness of the observed measurements to their true value

$$m_y = \sqrt{\frac{\sum_{i=1}^{n} {e_i}^2}{n}}$$

Co-funded by the
Erasmus+ Programme
of the European Union

ESTIMATION

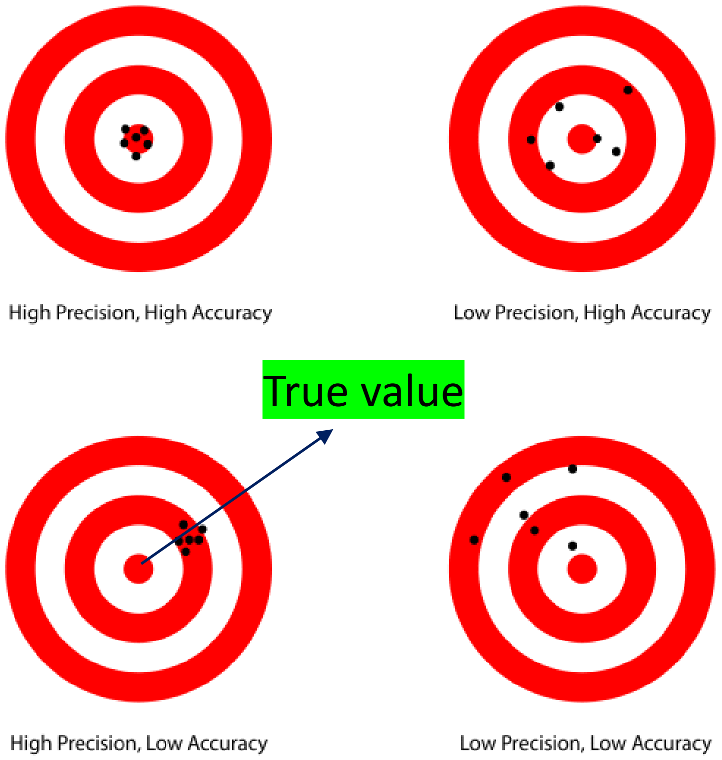


- A – Bias? Precise? Accurate?
- B – Bias? Precise? Accurate?
- C – Bias? Precise? Accurate?
- D – Bias? Precise? Accurate?

Is it possible to make precise and accurate from imprecise and inaccurate? Which one?

Co-funded by the
Erasmus+ Programme
of the European Union

Accuracy and Precision



Source: https://phidgets.wordpress.com/2014/05/20/accuracy-precision-and-resolution-theyre-not-the-same/

accuracy

The closeness of the measurement to the true value. For example, if we measure again the diameter with a measuring tape, we expect that the value that we will get will be close to the true value. Therefore the more the true value is exceeded, the less accurate the values will be and vice versa

precision

The degree of agreement in a series of measurements. For example if a tree diameter is measured 4 times and we got the same recorder value then we may say that these measurements were precisely measured

Co-funded by the Erasmus+ Programme of the European Union

Czech University of Life Sciences Prague Faculty of Forestry and Wood Sciences

Consistency

Following the same rules of measurement to each similar case encountered.

Example I: measuring trees always at 1.3 m above ground level

Example II: DBH acquisition in 4 different plots with caliper

- Try to be consistent using the same tool for all 4 plots
- It is inconsistent methodologically to alter between different tools

Example III: Size and geometry of the plot, etc. (maintain the same in all 4 plots)

Co-funded by the Erasmus+ Programme of the European Union

Bias - Unbiased Data

It is a statistics term which refers to the tendency of a sample statistic to systematically **over- or under-estimate** a population parameter

It can be as a result of many things. For example:

- ✓ instrumental error
- ✓ methodological error of the applicant either intentional or not, etc.

Bias often occurs when the sample (group of trees) does not accurately represent the population

Random sampling is a good solution

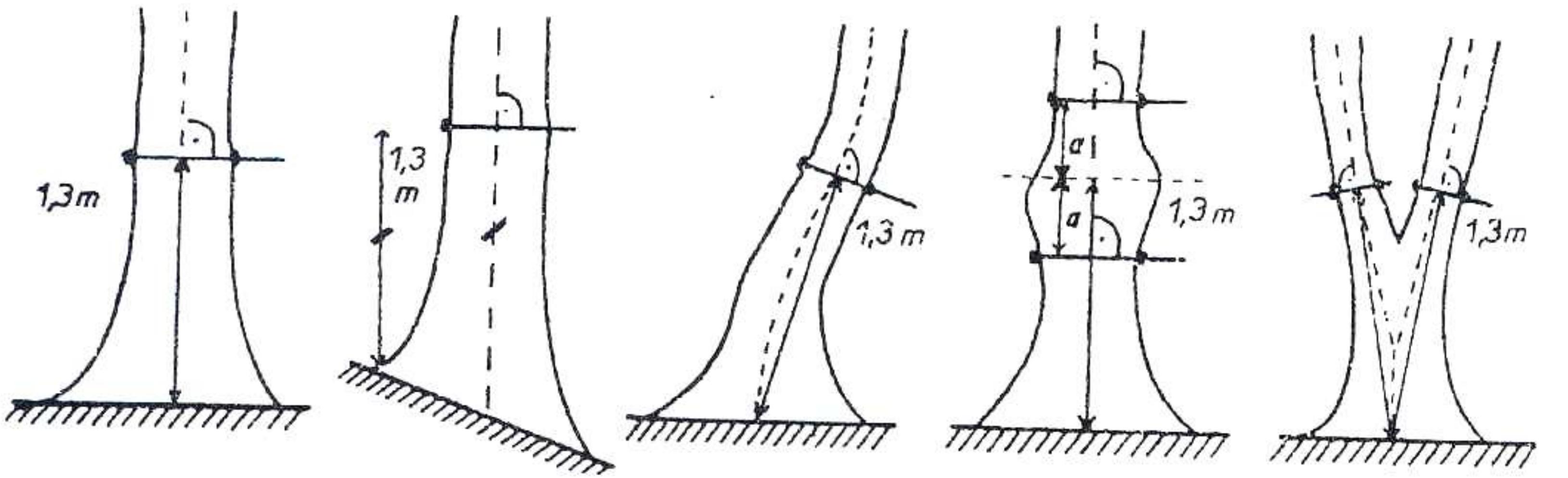
Let us imagine that we need to know what is the average height of a mixed and diverse forest stand?

?Which trees to measure?

Co-funded by the Erasmus+ Programme of the European Union

DIAMETER (d, DBH)

- usually over bark diameter at a fixed distance from the base of the tree – 1.30m or 4.5ft (1.37m); $d_{o.b.}$; $d_{u.b.}$



Co-funded by the
Erasmus+ Programme
of the European Union

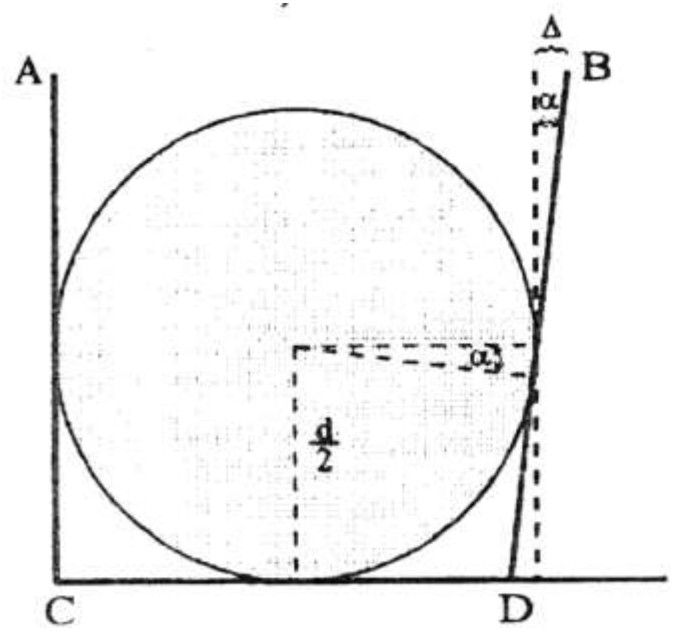
DIAMETER (d, DBH)

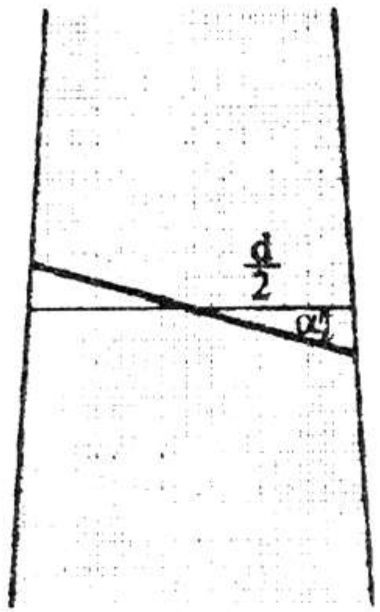
- Calipers

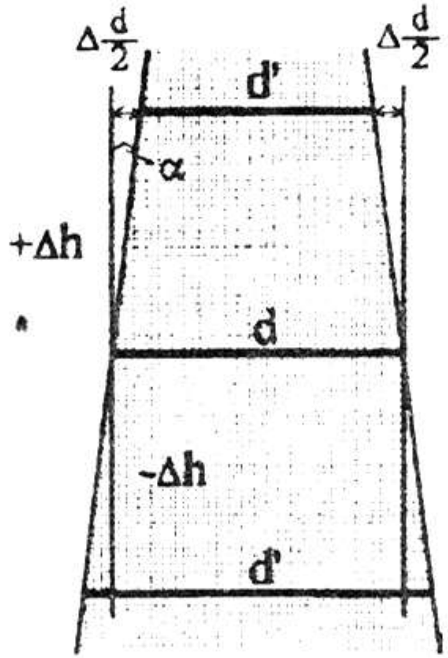
Co-funded by the
Erasmus+ Programme
of the European Union

DIAMETER (d, DBH)

- Measurement errors







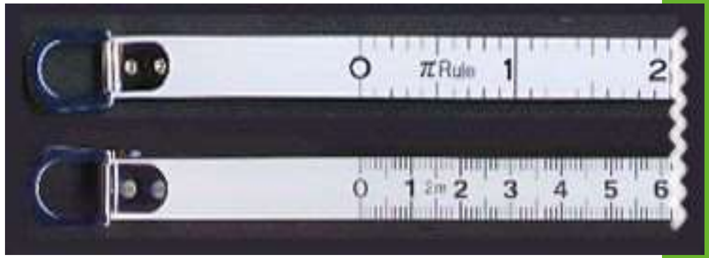
Co-funded by the
Erasmus+ Programme
of the European Union

BASAL AREA (g, BA)

- Cross-sectional area of the stem, either at breast height or at specified height above the base of the tree
- Derived from the tree diameter or from the stem circumference measured with a tape

$$BA = \frac{\pi}{4} d^2 \qquad \boldsymbol{m_g = 2m_d}$$

$$BA = \frac{C^2}{4\pi} \qquad \boldsymbol{m_{gc} = 2m_c}$$

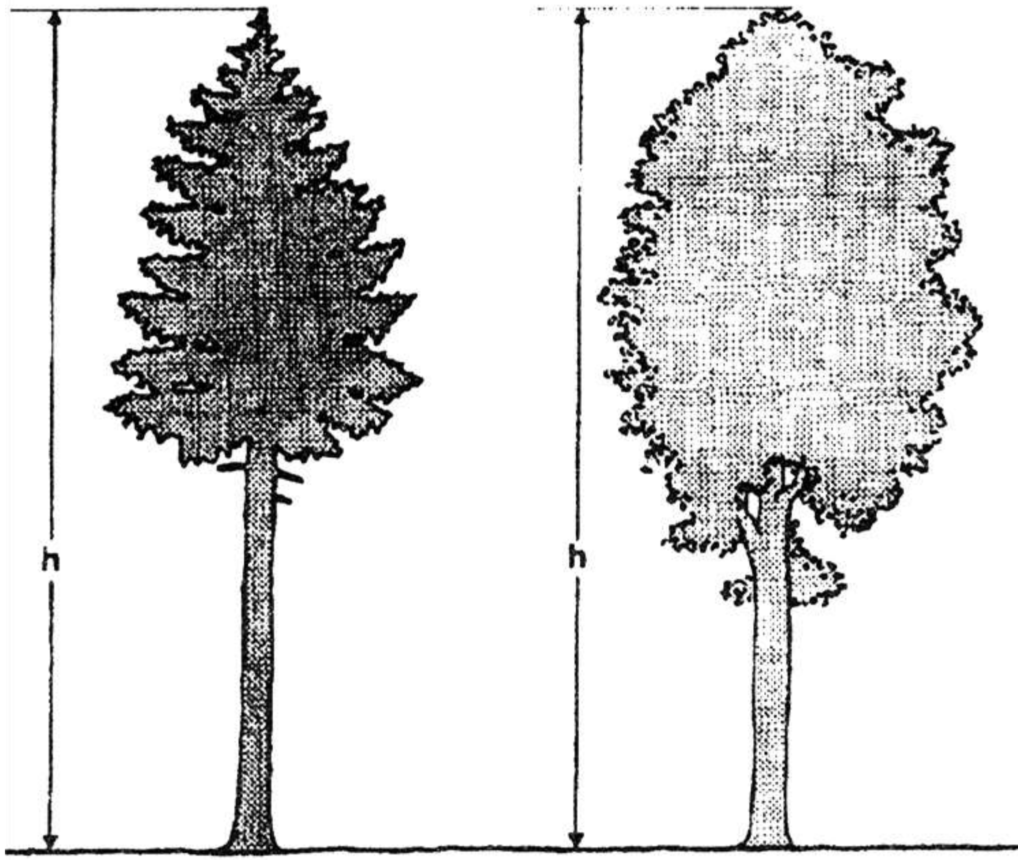


Co-funded by the Erasmus+ Programme of the European Union

FRAME

TREE HEIGHT (h)

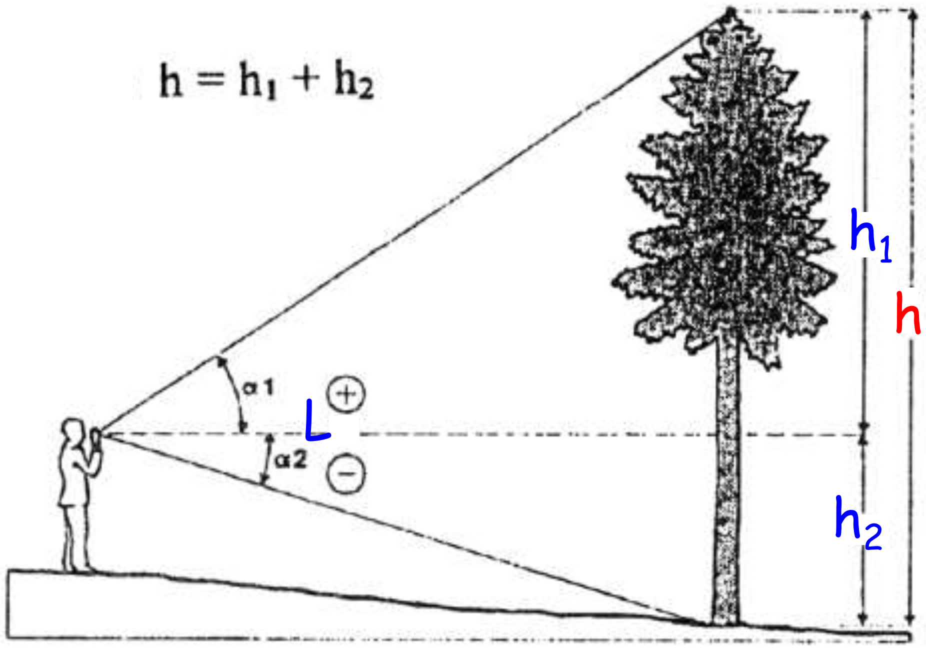
- Distance between the top and base of the tree, measured along a perpendicular dropped from the top.
- Merchantable height – upper point of measurement, which coincides with the limit of merchantability.

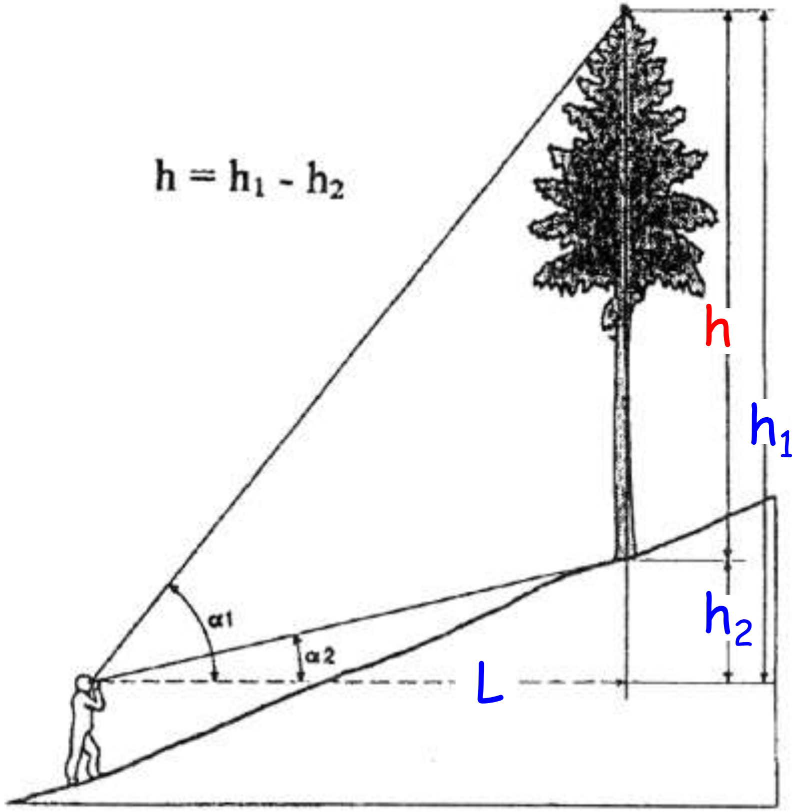


Co-funded by the Erasmus+ Programme of the European Union

TREE HEIGHT (h)

- Trigonometric principle





Measuring uphill

Co-funded by the
Erasmus+ Programme
of the European Union

TREE HEIGHT (h)

- Hypsometers

Vertex
(ultrasound)

Laser Vertex

Suunto

HEC

Silva

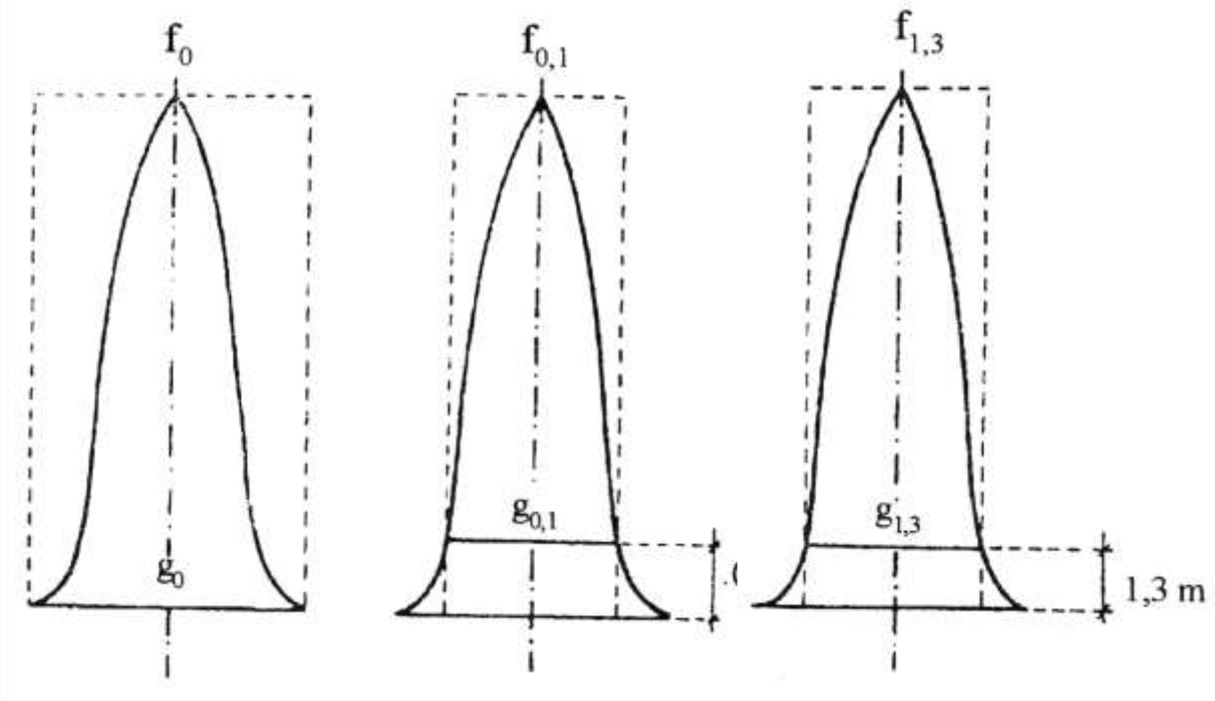
Co-funded by the Erasmus+ Programme of the European Union

FRAME

FORM FACTOR (f)

- Is definied as stem volume, expressed as a proportionn of the volume of a cylinder of the same height, with a diameter equal to the stem diameter at the selected reference point

$$f = \frac{stem\ volume}{cylinder\ volume}$$



Absolute True False

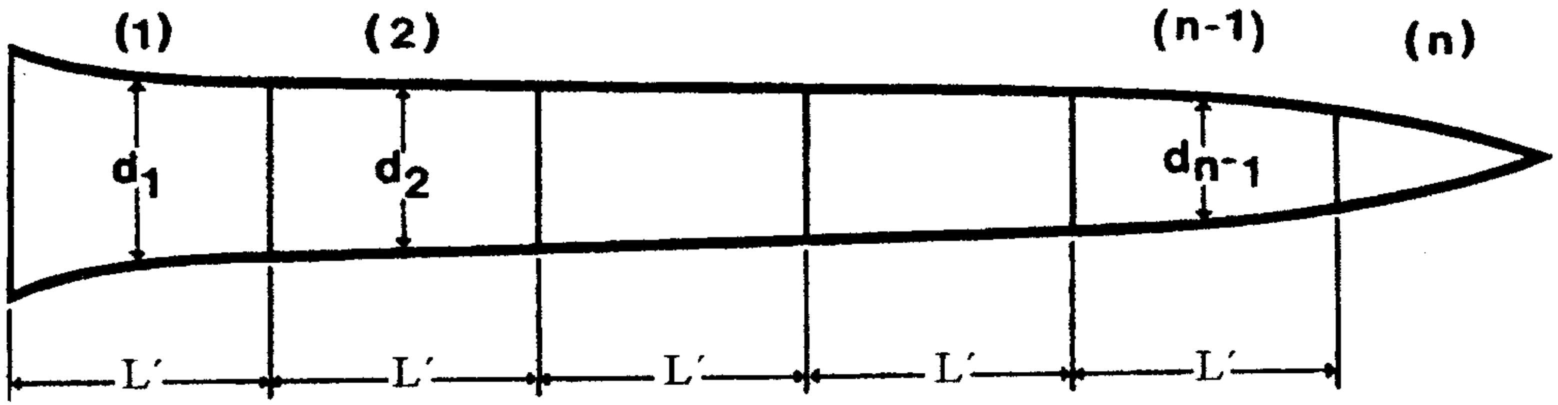
Co-funded by the Erasmus+ Programme of the European Union

ROUNDWOOD VOLUME (v)

- to estimate parameters of volume equations and to construct volume tables

$$v = g * l = \frac{\pi}{4} d^2 * l$$

$$v = v_1 + v_2 + \cdots v_{n-1} + v_n$$



$$v = L^{\prime}(g_1 + g_2 \cdots g_{n-1}) + (g_n * l_t)$$

$$v = \frac{\pi}{4} L^{\prime}(d_1^2 + d_2^2 \cdots d_{n-1}^2) + \frac{\pi}{4} d_n^2 * l_n$$

Co-funded by the
Erasmus+ Programme
of the European Union

ROUNDWOOD VOLUME (v)

Huber – cross-sectional area at the midpoint

$$v = g_m * l$$

Smalian – cross-sectional area at the lower and upper end

$$v = \frac{g_u + g_l}{2} * l$$

Co-funded by the
Erasmus+ Programme
of the European Union

FRAME

STACKED WOOD VOLUM (v)

- Volume is determand conversion factor applied to adjust for a free space between the roundwood logs (for example 1x1x1m = approx. 0.60-0.70m^3)
- photos
- weight

$$v = w * \rho$$

for example 1 tonne = 420 – 500 kg/m^3 depending on the moisture (water content) and age.

Co-funded by the Erasmus+ Programme of the European Union

TREE VOLUME (v)

- Volume
  - stem volume, total tree volume (including branches), merchantable volume
  - over or under bark

$$v = g * h * f$$

- estimation

$$v = 0.785d^2 * h * 0.45$$

Co-funded by the
Erasmus+ Programme
of the European Union

FRAME

TREE VOLUME TABLES AND EQUATIONS

- number of entries and predictor variables of the volume function
  - single-entry volume function – **dbh**
  - two entries – **dbh and height**
  - more entries – **dbh, height + entry X** (diameter at 30% of the height, height above ground of the base of the life crown, etc.)

$$v = a * DBH^b$$

$$v = a + b * DBH^2 * h$$

$$v = a * (DBH^2 * h)^b$$

$$v = a * DBH^b * h^c$$

Merchantable volume proportion V5/V

STAND VOLUME (V)

- Whole stand calipering
- Sampling - representative methods
- Mean tree volume
- Yield tables
- Estimation

Co-funded by the Erasmus+ Programme of the European Union

STAND VOLUME (V)

Whole stand calipering

- DBH measurement of all trees
  - DBH classes (2cm, 4cm, etc.)
  - Individual DBH – electronic caliper
- Height of samples (approx. 5 per dbh class)
  - Strong correlation between height and DBH
  - Time consuming measurement

Co-funded by the Erasmus+ Programme of the European Union

STAND VOLUME (V)

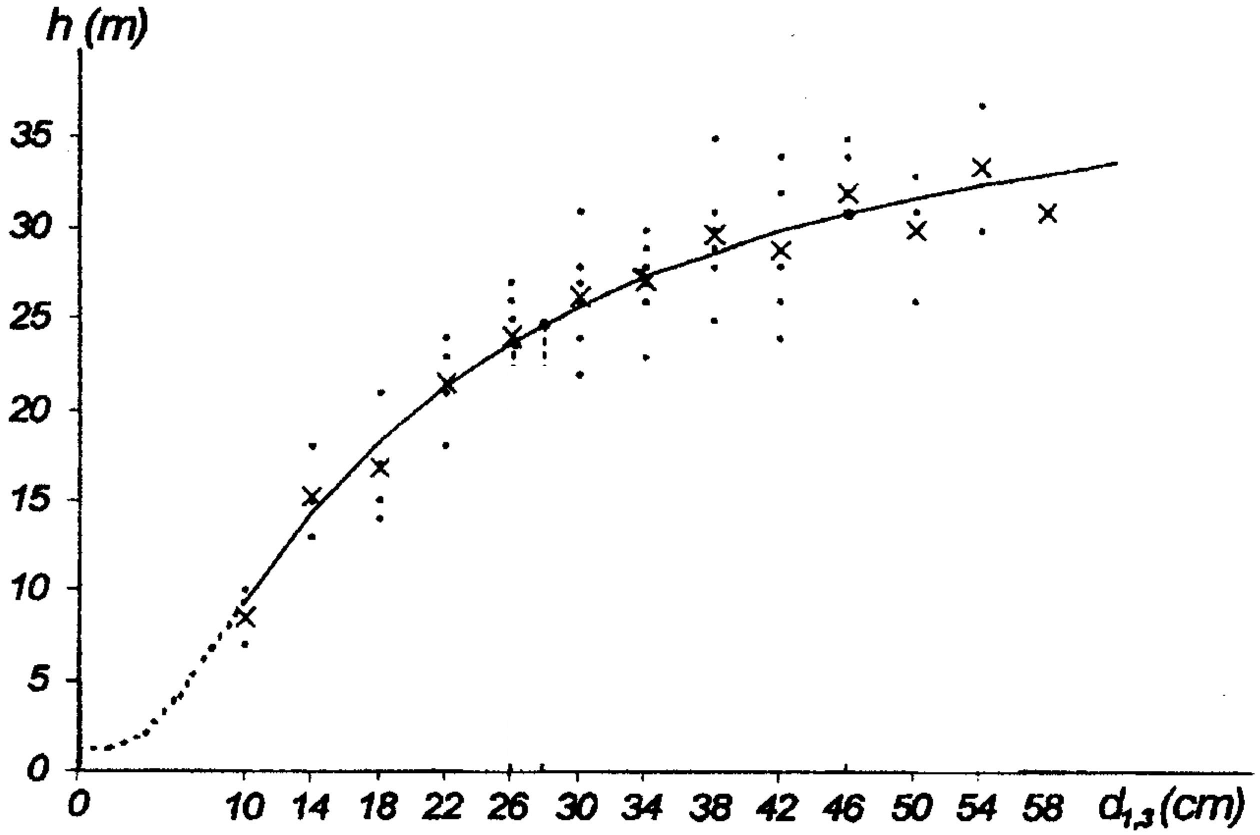
FRAME

Height curve fitting

$$h = 1.3 + a * e^{DBH/d}$$

$$h = 1.3 + \frac{DBH^2}{b_0 + b_1 * DB + b_2 * DBH^2}$$

$$h = b_0 + b_1 * ln(BDH)$$



Co-funded by the
Erasmus+ Programme
of the European Union

STAND VOLUME (V)

Volume of individual tree

- DBH and fitted height
- Using volume equation or volume tables

Stand volume = sum of volume of individual trees

- Most precise method
- Electronic devices, softwares

Co-funded by the Erasmus+ Programme of the European Union

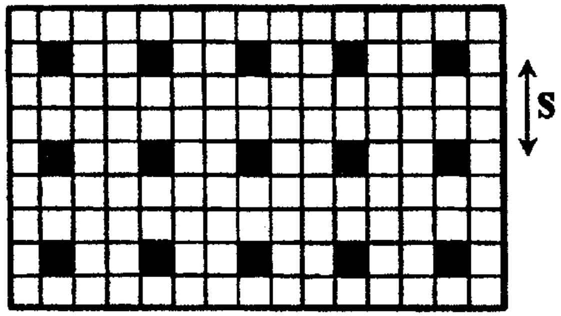
STAND VOLUME (V)

- Sampling
    - Especially for large forest units - to spare time and money
    - Effective in evaluation of development
- Consists of *n* sampling units on which tree are measured or estimated

N = 135; n = 15 (I = 11 %)

$$n = \frac{t_{\propto}^{2} * \sigma_{x}^{2}}{\Delta_{\bar{x}}\%}$$

$$\mu - \bar{x} = \Delta_{\bar{x}}\%$$



t_α – reliability coeficient (1.96)
σ_x – variability
Δ_x – accetable error

$$V = V_{SP}\frac{100}{I\%}$$

Co-funded by the Erasmus+ Programme of the European Union

STAND VOLUME (V)

FRAME

Sampling methods

- plot sampling
- point sampling
- multistage sampling
- ...

Plot size

- given radius (r = 7, 10, 13m ...)
- given area (a = 100, 500m^2, ...)
- Optimal size produce higher precision for a given costs

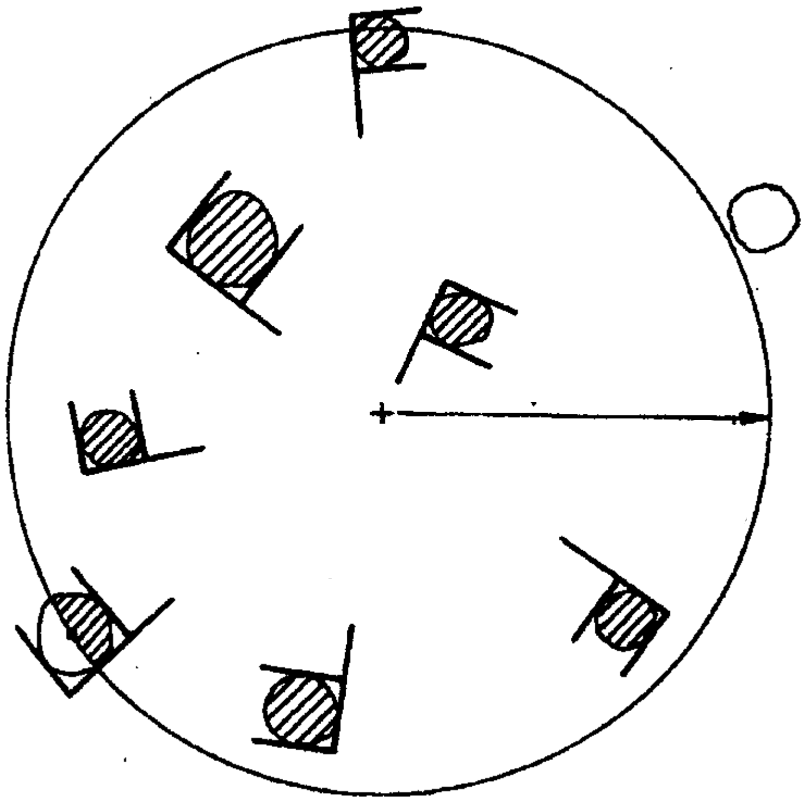
Plot shape

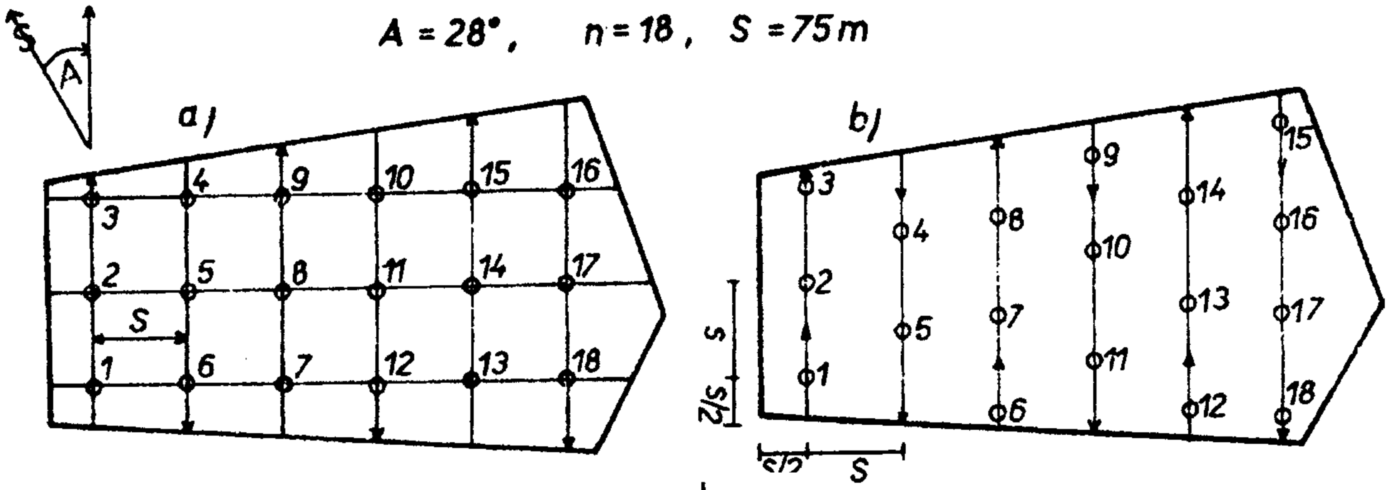
- circular
  - smallest perimeter for a given plot size
  - no right angles (one man work
  - plot boundaries located by optical devices (Vertex)
- square (research)
- rectangular (plantation)

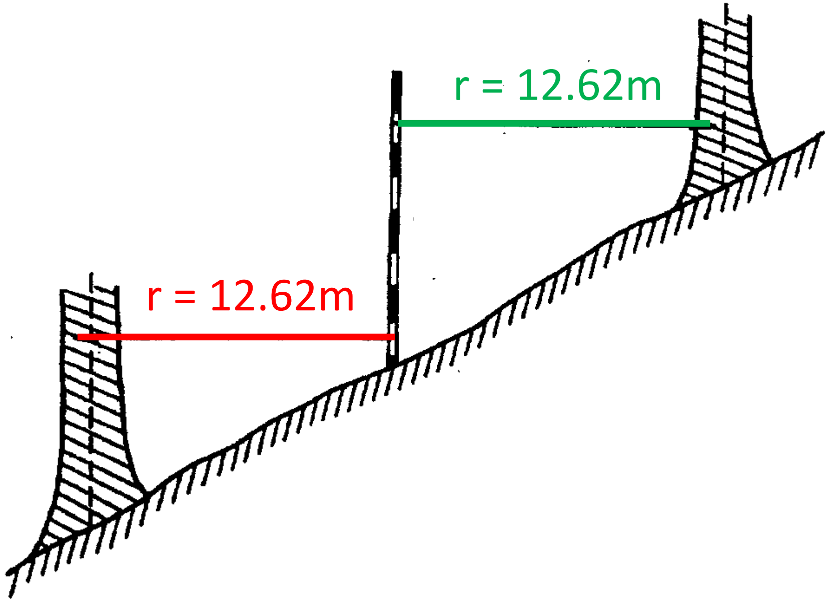
Co-funded by the Erasmus+ Programme of the European Union

STAND VOLUME (V)

- Plot sampling



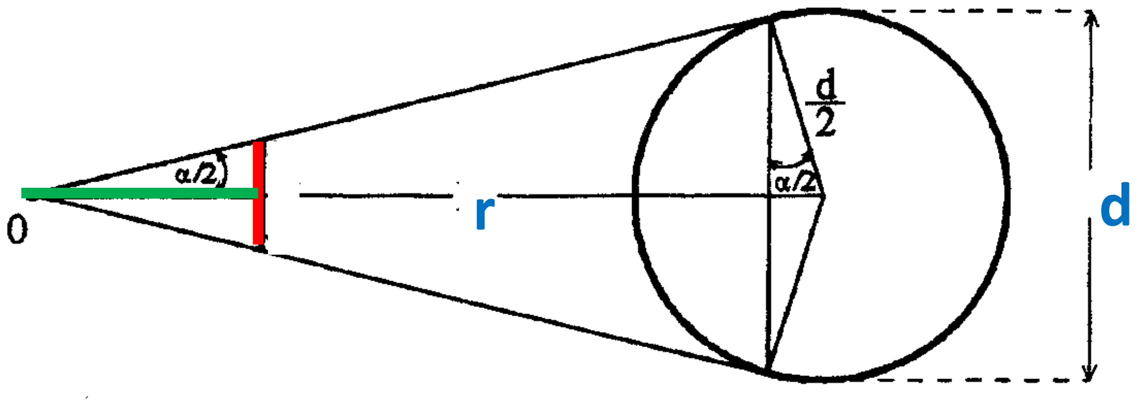




Co-funded by the Erasmus+ Programme of the European Union

STAND VOLUME (V)

- Point sampling (angle count sampling, relascope sampling)
- Imaginary plot boundaries



A rod with a length of ***c*** units and cross-arm (blade) of **1** unit

$$\frac{d_i}{r_i} = \frac{1}{c}$$

STAND VOLUME (V)

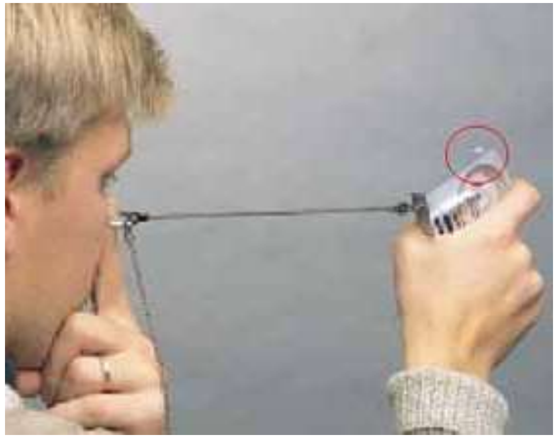
Co-funded by the Erasmus+ Programme of the European Union

- Tree counting
  - IN = 1
  - Boundary = 1/2
  - OUT = 0

IN OUT BOUNDARY

- Volume

$$V = G * H * F$$



Co-funded by the Erasmus+ Programme of the European Union

FRAME

MEAN DIAMETER, MEAN HEIGHT

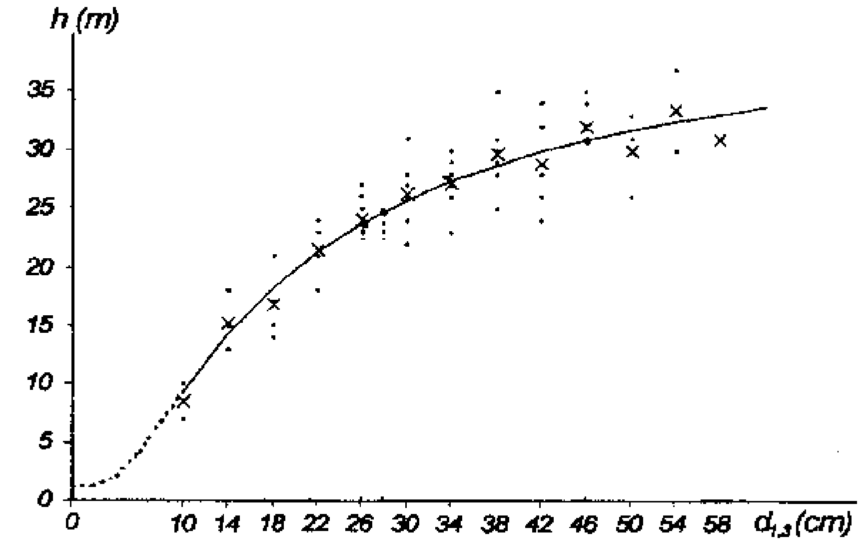
- Quadratice mean diameter

$$d_g = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}}$$

for gouped data

$$d_g = \sqrt{\frac{\sum_{i=1}^{k} n_i * d_i^2}{\sum_{i=1}^{k} n_i}}$$

- Mean height – height of the tree with the quadratice mean diameter – derived from the height curve



Co-funded by the Erasmus+ Programme of the European Union

STAND DENSITY

- Current stocking (basal area) per hectare expressed as a percentage of volume (basal area), which is considered as a „normal“ per hectare for a given species, age, thinning regime, ...

$$\rho = \frac{V_{real}}{V_{model}} = \frac{G_{real}}{G_{model}}$$

- Ratio of current and model number of trees (plantation)

$$\rho = \frac{N_{real}}{N_{model}}$$

Co-funded by the Erasmus+ Programme of the European Union

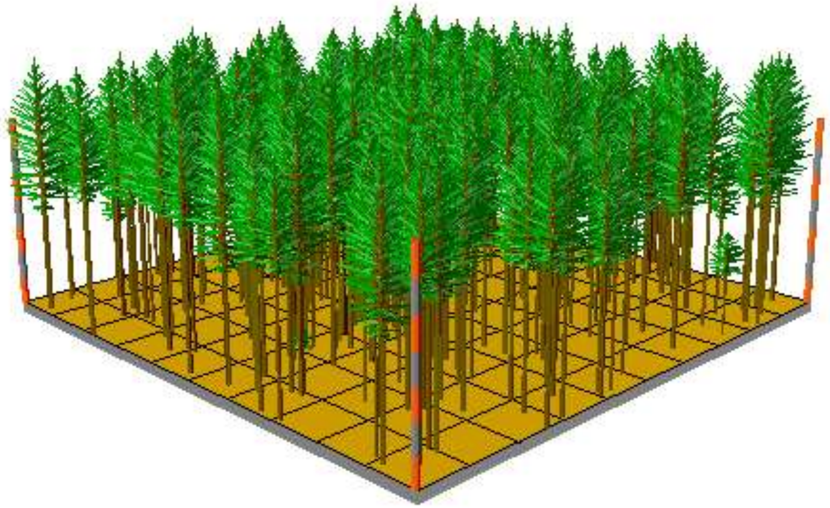
Stand Density

FRAME

Czech University of Life Sciences Prague Faculty of Forestry and Wood Sciences

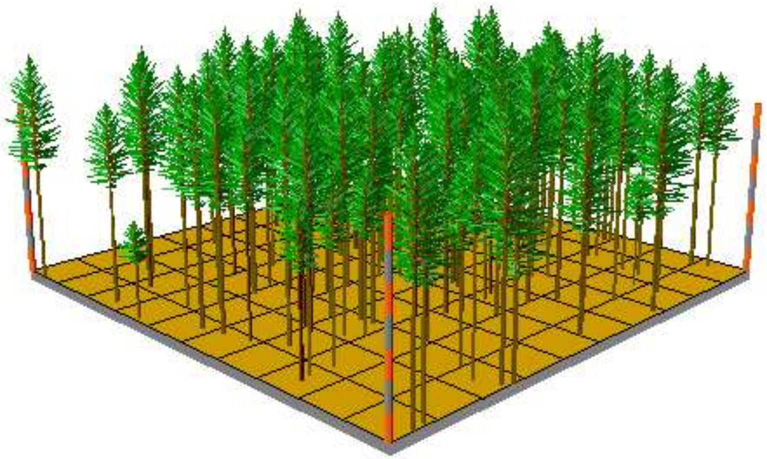
Stand Density: A quantitative measure of the degree of crowding or competition existing within the stand

Also known as **Reineke's Stand Density Index** after its founder. It is a measure of the stocking of a **stand** of trees based on **the number of trees per unit area and quadratic mean diameter.**



Example of high density

Example of lower density



Co-funded by the Erasmus+ Programme of the European Union

Stocking

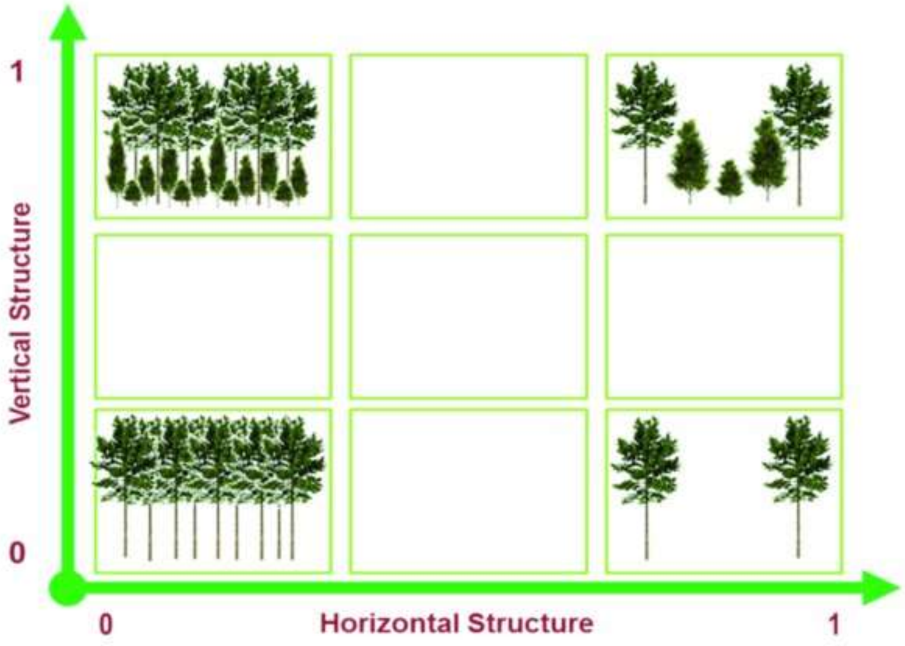
Stocking refers to the adequacy of a given stand density to meet some specified management objective. Hence, stands are often referred to as **understocked, fully stocked, or overstocked**.
Stocking is a relative concept - a stand that is overstocked for one management objective may be understocked for another

In addition stocking is further modified and defined as:

- **Fully stocked stands** – Stands in which all the growing space is effectively occupied but which still have ample room for development of the crop trees (in other words - when the trees utilize the available growing space)

- **Overstocked stands** – Stands in which the growing space is so completely utilized that growth has slowed down and many trees, including dominants, are being suppressed (no more capacity)

- **Understocked stands** – Stands in which the growing space is not effectively occupied by crop trees



Source: Victor Cazcarra-Bes et al. 2017

Competition is of two types:

1. **Intraspecific** (individual of the same species compete for limited resources)
2. **Interspecific** (individuals of *different* species compete for the same resources in an ecosystem)

✓ Whatever the structure of forests**, rate of growth depends on the degree of competition**

Co-funded by the
Erasmus+ Programme
of the European Union

FRAME

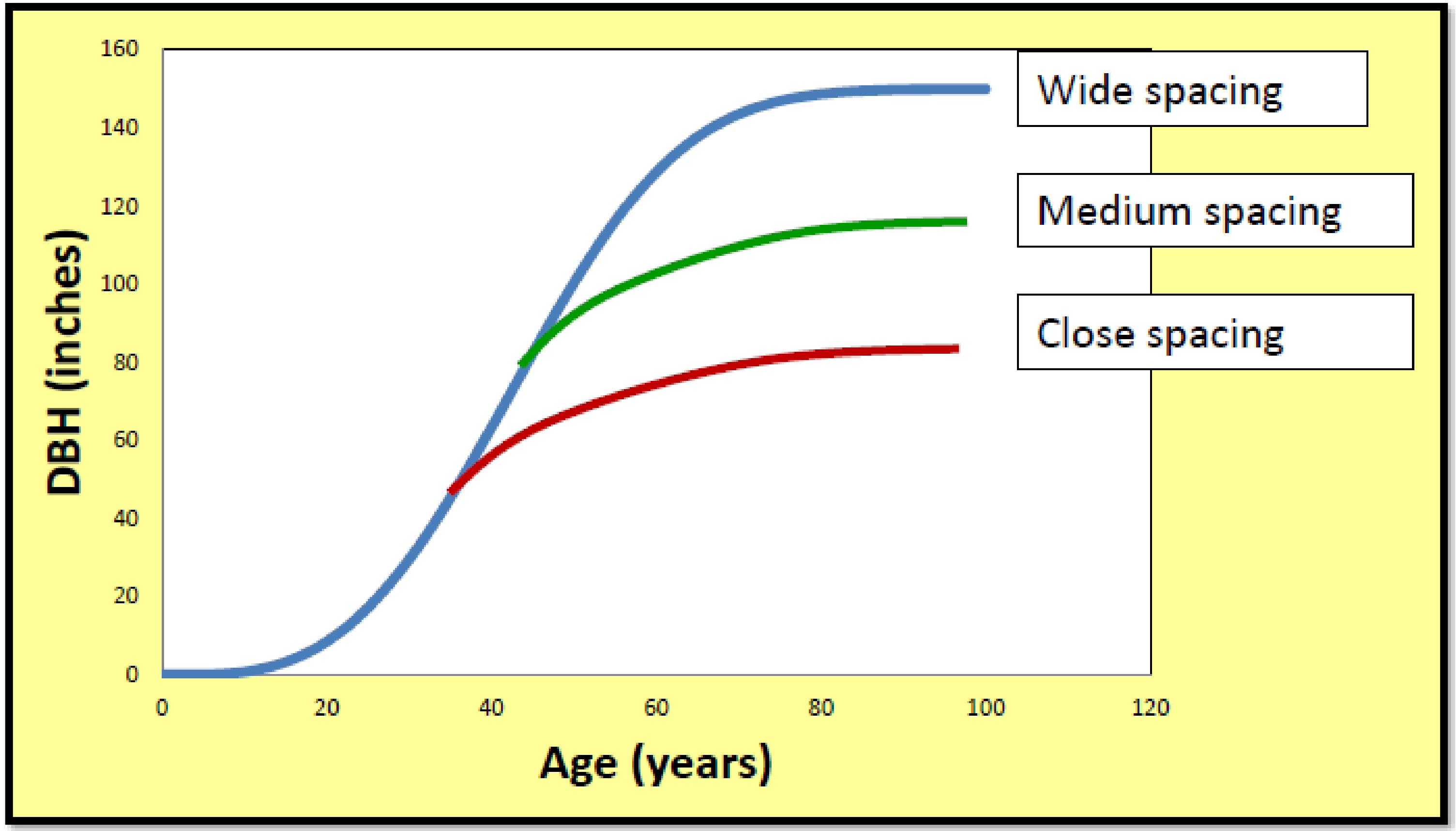
Competition Effect on Stand Density

Competition between trees within a stand has the following effects:

- it **reduces height growth rates** (only for some species)
- it **reduces diameter growth rates**
- it **increases mortality rates**
- it **increases crown recession rates** (refers to crown radius/diameter)

Co-funded by the Erasmus+ Programme of the European Union

Stand Density Effect on Diameter Growth



Co-funded by the
Erasmus+ Programme
of the European Union

GROWTH

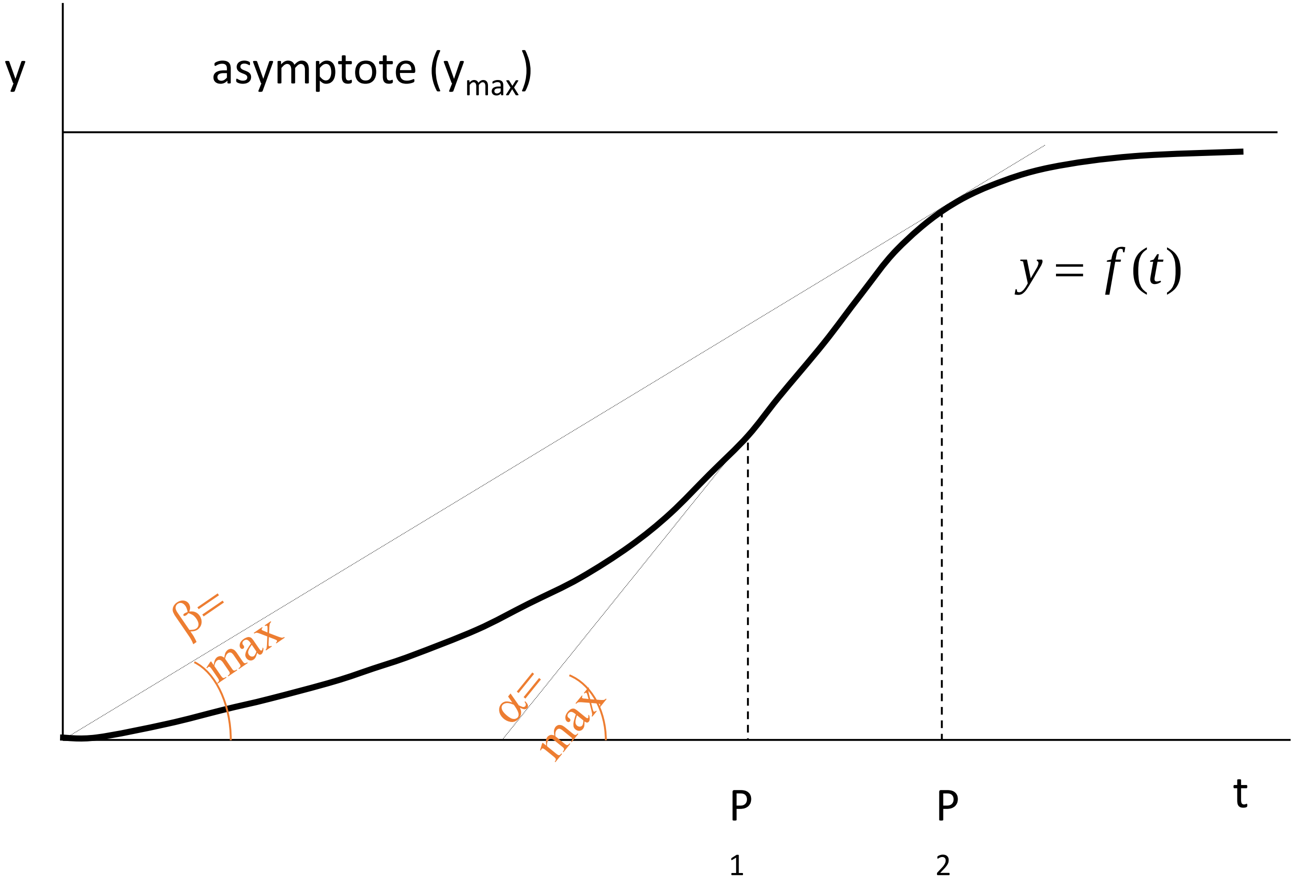
- enlargement of dimension of live system by assimilation activity (Bertalanfy 1951)
- growth values y (parameters) can growth
  - t – age
  - U – environment (water, precipitation, temperature, nutrients, CO_2, …)

$$y = \mathrm{f}(t, U) \quad \Rightarrow \quad y = \mathrm{f}(t)$$

GROWTH

Growth curve

- sustainable increasing (no decline) – at least monotonous
- S - shape
- asymptotic behaviour
  - when t = 0 then y = 0
  - when t = tmax then y = ymax
- at least one inflexion point



GROWTH

- Korf (1939)

$$y = A * e^{\frac{k}{(1-n)t^{n-1}}}$$

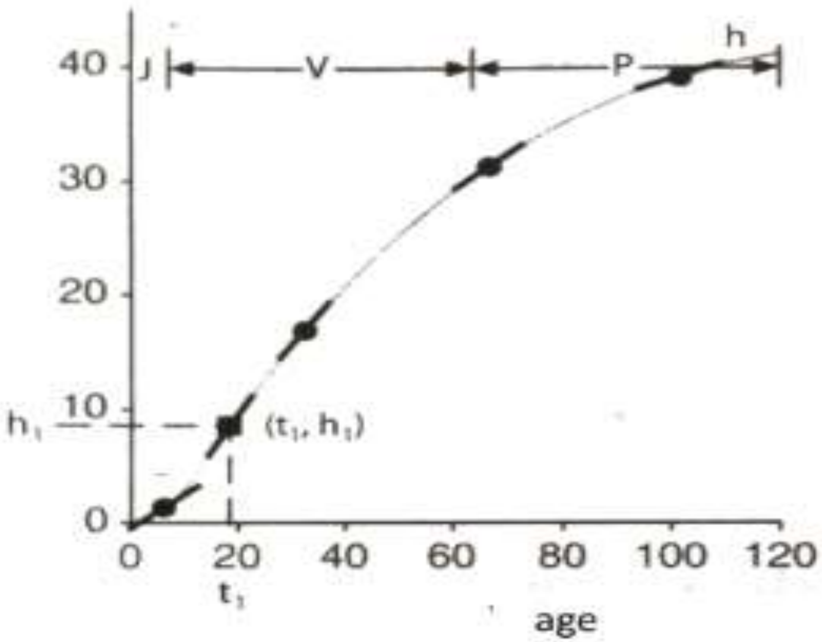
- Richards & Chapmann (1959)

$$y = y_{\max} * e^{a\left(1 - e^{\frac{c}{1-m}t^{-m}}\right)}$$

Co-funded by the
Erasmus+ Programme
of the European Union

Definition of Increment

<u>Increment or accretion</u> refers to the **quantitative increase in size in a specified time interval due to growth**



First derivation of growth function

- enlargement of growth value (i_y)

i_y can be referred to:

- ✓ diameter
- ✓ height
- ✓ basal area
- ✓ volume

Co-funded by the Erasmus+ Programme of the European Union

FRAME

Measurement of Increment

Method of Yield Table

PROS

- if yield tables are available for a territory , then future estimations of increments on even-aged forest are possible.
- Necessary knowledge of site index, density, age.

Cons

- Yield tables are constructed for forest stands with 100 % (fully stocked) (in reality that doesn't happen)

Method of Continuous Forest Inventory

PROS

- Statistical methods based on permanent fixed area sample plots
- Empirical measurements with an interval of 5-10 years, depending on the country and forest management plans (FMP)

Cons

- Consistency of data collection might be challenging because of the involvement of different people

Co-funded by the Erasmus+ Programme of the European Union

Faculty of Forestry and Wood Sciences

Factors Affecting Increment

The increment varies with:

- ✓ Internal conditions (i.e., genotype)
- ✓ External conditions (i.e., geologic/edaphic, climatic etc.)
- ✓ Tree species

Co-funded by the
Erasmus+ Programme
of the European Union

Faculty of Forestry and Wood Sciences

FRAME

Categories of Increment

Current Increment

According to time period

- annual (CAI)
- periodic (CPI)
- total (CTI)

Difference of value (***y***) in different times – time period (***t_1–t_2***)

Mean Increment

According to time period

- annual (MAI)
- final (MFI)

Divide of value (y) and time period (n) when this value has grown

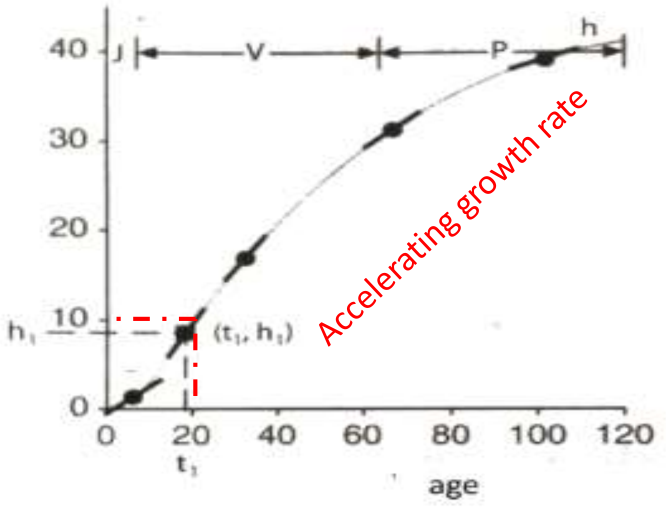
Current Annual Increment

CAI: It is the growth during one (last) year

$$CAI = y_t - y_{t-1}$$

For example, the height of the tree in *t-1 (19)* was 9 m. In *t (20)* is 10 m. What is the CAI?

$$CAI = 10 - 9 = 1 \text{ m/year}$$



Co-funded by the Erasmus+ Programme of the European Union

Periodic Annual Increment

- PAI is the change in the size of a tree between the beginning and ending of a growth period, divided by the number of years that was designated as the growing period
- Increases rapidly and then quickly declines, approaching zero
- **Periodic annual increment (PAI)** is commonly used instead of **current annual increment (CAI)** as a basis for computing growth *percent*.
- PAI may go negative if a tree loses volume due to damage or disease.

For example, lets say that the growth period is from age 5 to age 10, and the yield (h), is 14 m at the beginning of the period and 34 m at the end. Calculate the PAI

$$PAI = \frac{34 - 14}{10 - 5} = 4 \text{ m/year}$$

$$PAI = \frac{y_2 - y_1}{t_2 - t_1}$$

y_2: yield at the ending of growing period
y_1: yied at the beginning of growing period

t_2: is the end year of the growth period
t_1: represents the year starting the growth period

Co-funded by the Erasmus+ Programme of the European Union

Current Periodic Increment

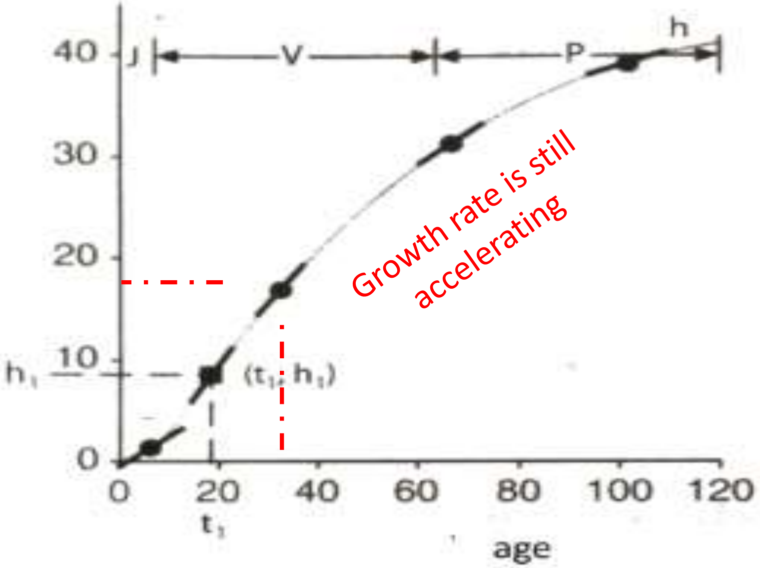
FRAME

Faculty of Forestry and Wood Sciences

CPI: It is the growth during some time period (5 or 10 years; in general ***n*** years)

- ✓ Similar as before but this time the difference is not per one year but for a period of time (n-years)

$$CPI = y_t - y_{t-n}$$



For example, the height of the tree in *t-10 (20)* was 9 m. In *t (30)* is 17 m What is the CPI?

$CPI =$ **17 – 9 = 8 m/10 years**

Co-funded by the
Erasmus+ Programme
of the European Union

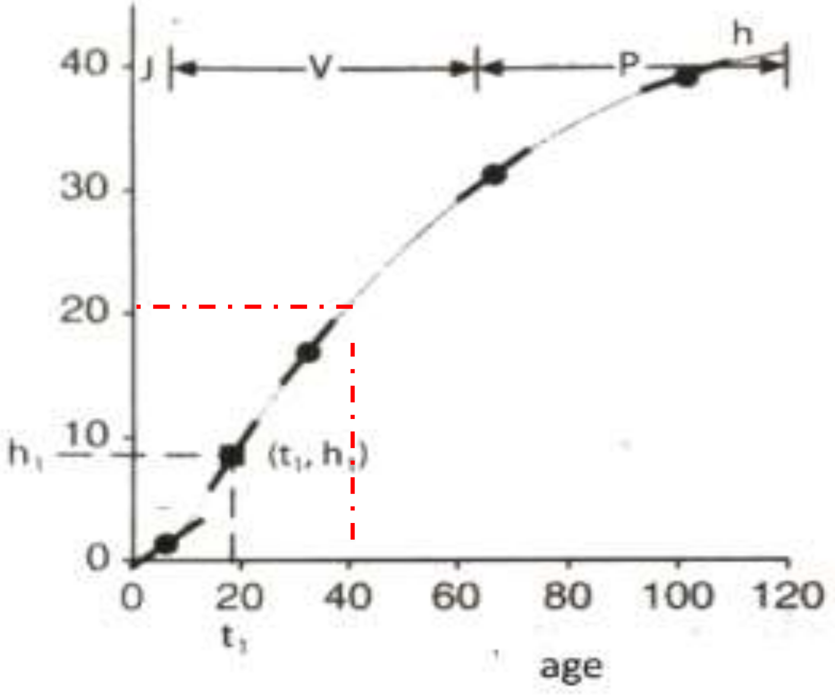
Current Total Increment

CTI: It is the growth during the whole growth period (life) from age ***0*** to age ***t*** (present age)

$$CTI = y_t - 0 = y_t$$

For example, the height of the tree in *age 0 was 0.* In *t (40)* is 20 m What is the CTI?

$$CTI = 20 - 0 = 20 \text{ m}$$



Growth rate in this phase start becoming constant (the exact period is also relative based on other parameters that will determine that i.e., species, competition etc.

Co-funded by the Erasmus+ Programme of the European Union

Mean Annual Increment

Czech University of Life Sciences Prague
Faculty of Forestry and Wood Sciences

FRAME

Mean annual increment (MAI) or mean annual growth refers to the average growth per year a tree or stand of trees has experienced to a specified age

$$MAI = \frac{y_t}{t}$$

$y_t \rightarrow$ Yield at time (t)

$t \rightarrow$ Examined time (t)

- ✓ Because the typical sigmoidal growth patterns of most trees, the MAI starts out small, increases to its maximum value as the tree matures, then declines slowly over the remainder of the tree's life.
- ✓ The MAI always remains positive.

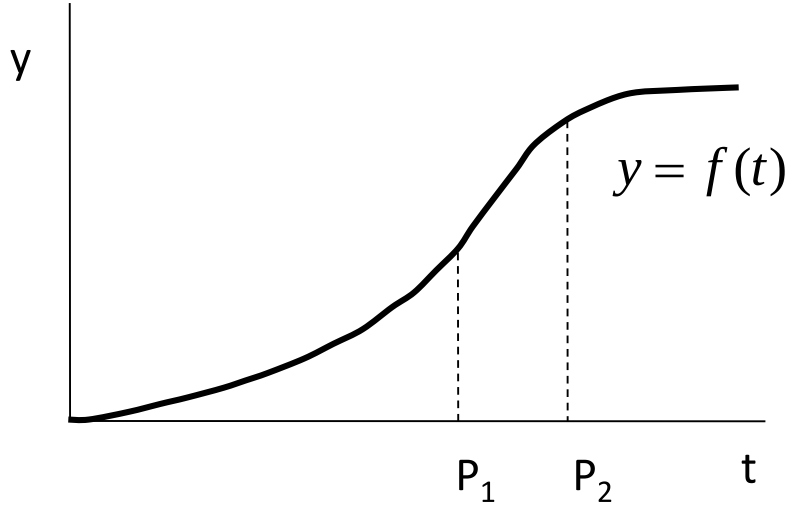
For example, a 20 year old tree that has a DBH of 10.0 cm. What is the MAI;

$$\boldsymbol{MAI = \frac{10}{20} = 0.5\ cm/year}$$

Co-funded by the Erasmus+ Programme of the European Union

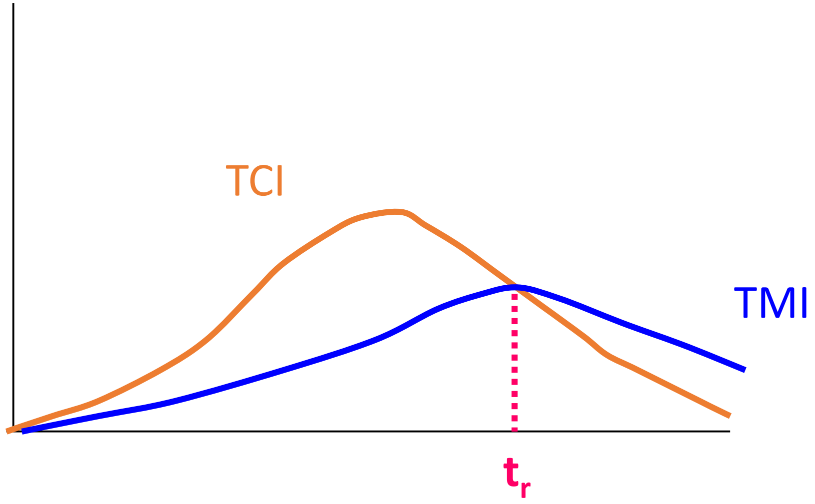
INCREMENT

Total volume production (TVP) – the total production of timber volume from a forest stand from the time of establishment up to a given age



$$TCI = \frac{y_t - y_{t-n}}{n} = \frac{TVP_t - TVP_{t-n}}{n}$$

$$TMI = \frac{y_t}{t} = \frac{TVP_t}{t}$$

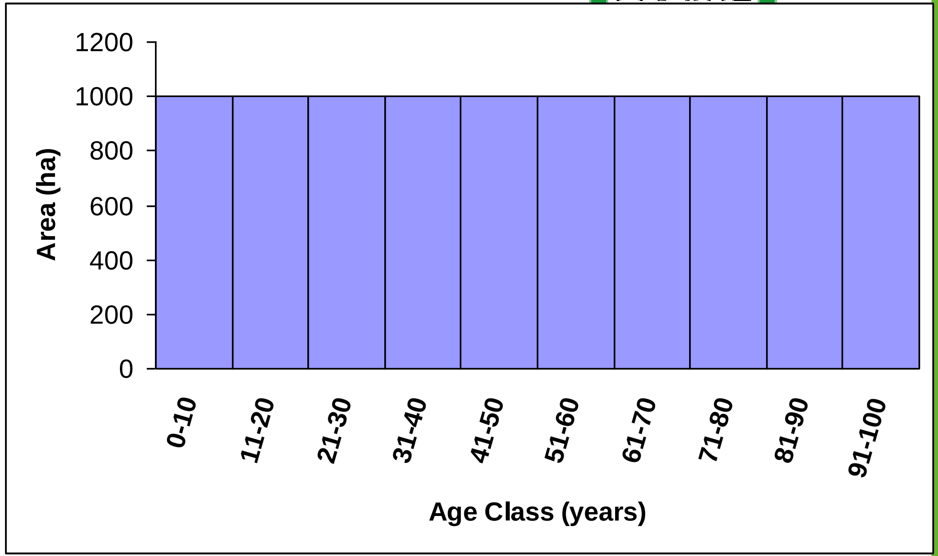


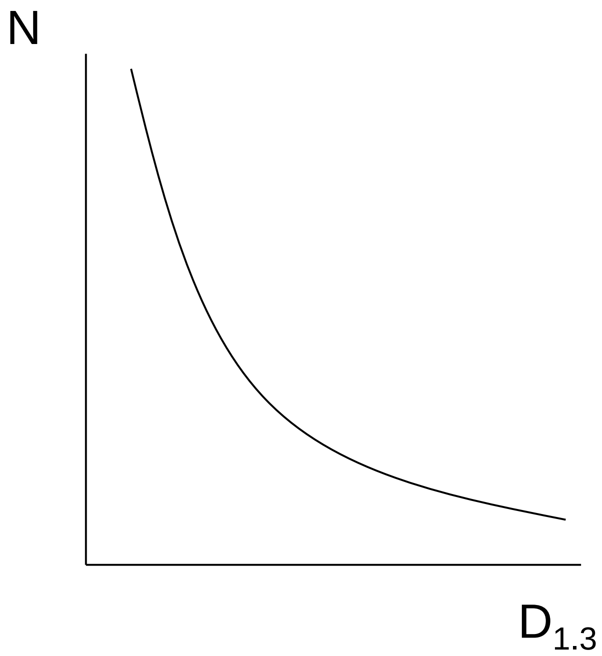
FOREST PLANNING

Forest models

- Normal (regulated) forest
- Selection forest

- The idea is to secure balanced and sustainable harvest for a long time
- Cut = increment



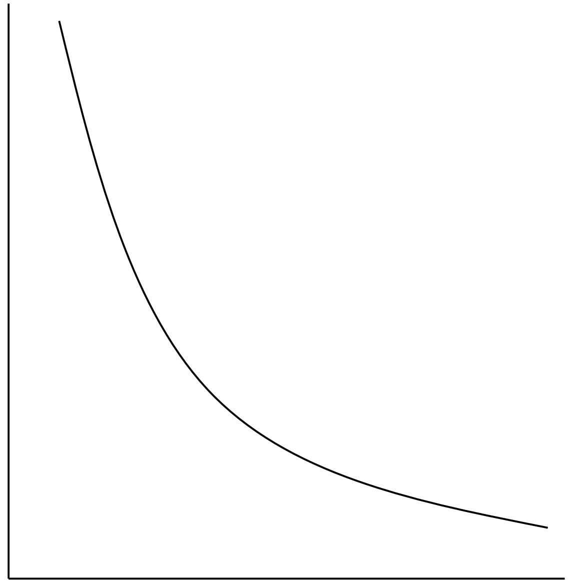


Co-funded by the Erasmus+ Programme of the European Union

FOREST PLANNING

Selection forest

- DBH distribution, no age
- Target diameter, no rotation age
- Permanent ingrowth (I)
- Inventory on permanent plots



$$TCI = V_t - V_{t-n} + Cut - I$$

Co-funded by the Erasmus+ Programme of the European Union

DECISION METHODS

FRAME

To find optimal solution:

- Maximise production or minimise costs
- Take into acount constraints

LP models

- very general optimization technique
- designed and used primarily to solve managerial problems
- applied to many different problems inclusive forest planning

Buongiorno J., Gilless J., 2003: Decision Methods for Forest Resource Management. Academic Press

Co-funded by the Erasmus+ Programme of the European Union

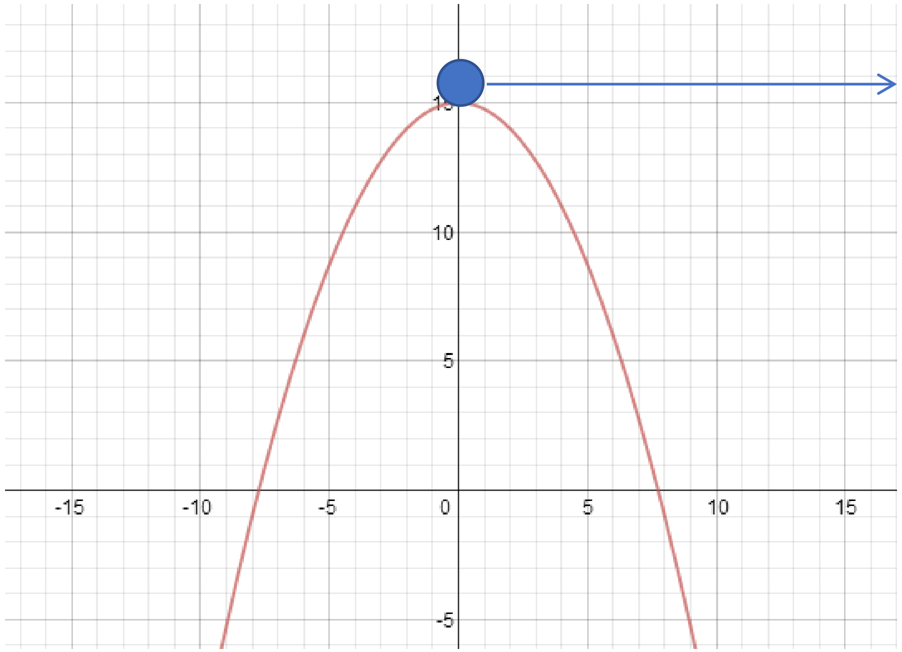
Linear programming

- Basic optimization tool, mathematical method
- Achieve the best result possible and meet the conditions
- „Linear“
  - Only linear variables in the model
  - Nothing like: $x^2, \sqrt{x}, \log(x), \frac{1}{x}$...
- „Programming“
  - Not a computer programming
    - Computer can help however
  - Create an optimal programme (=schedule, planning)
    - Forest harvesting
    - Crop rotation
    - Mixture making
    - Shift planning
    - Material cutting

Co-funded by the
Erasmus+ Programme
of the European Union

How does it work?

- We are searching for an optimal solution
  - Optimal solution = extreme point (max/min) of the OBJECTIVE function
- Bounded vs. unbounded extremes



MAX (unbounded)

Co-funded by the Erasmus+ Programme of the European Union

Search space

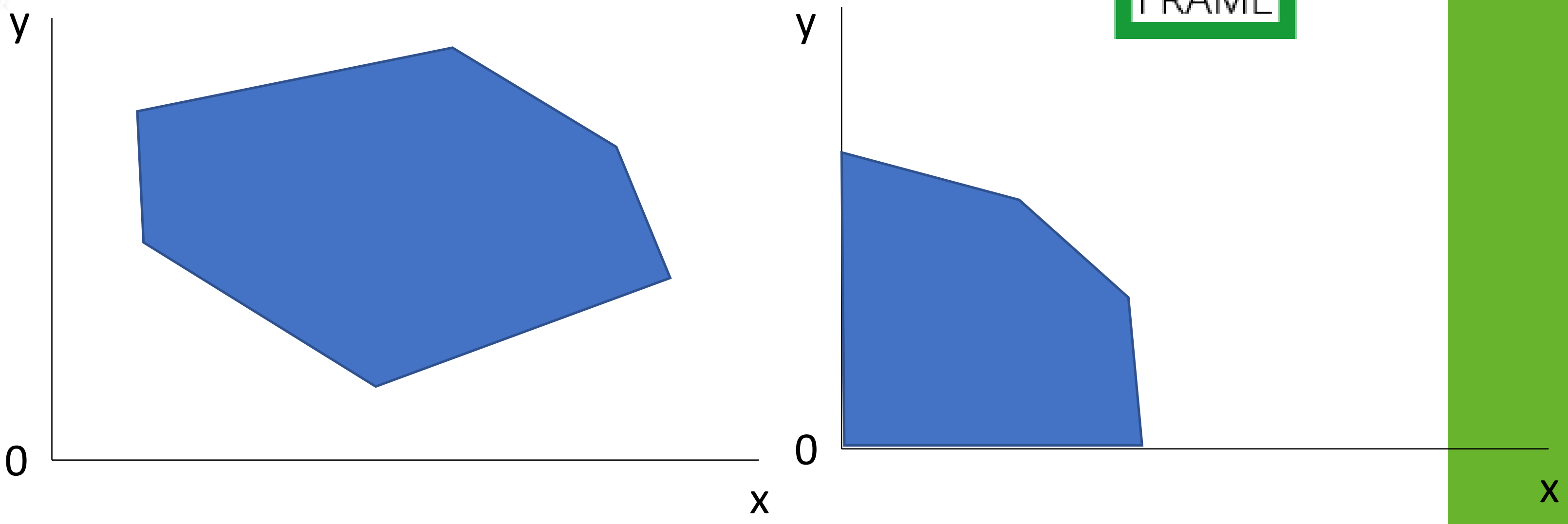
- Bounded extreme
- We are searching for an optimal solution ONLY in particular region
- Region given by constraints
  - Constraints = inequalities (usually)
  - Example: $x_1 + x_2 \leq 10$
- Region = feasibility region
  - Conditions (=constraints) are met
- Has to be convex!
- Has to be nonnegative!

Co-funded by the
Erasmus+ Programme
of the European Union

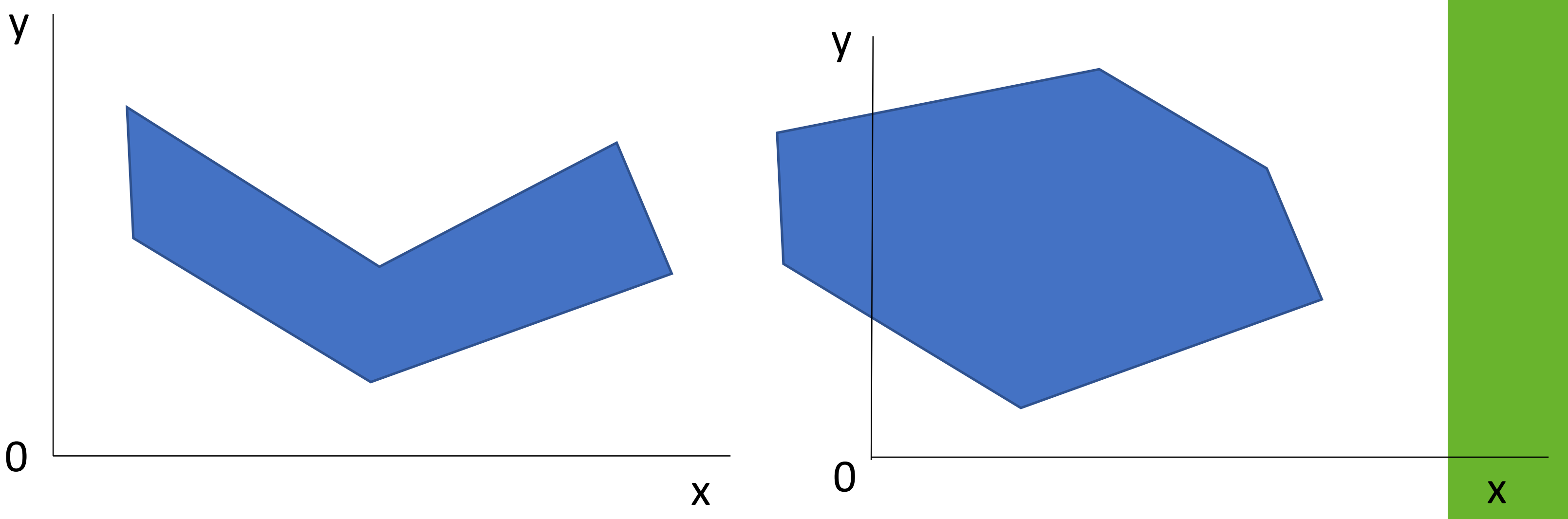
Feasibility region – examples

FRAME

- Feasible:



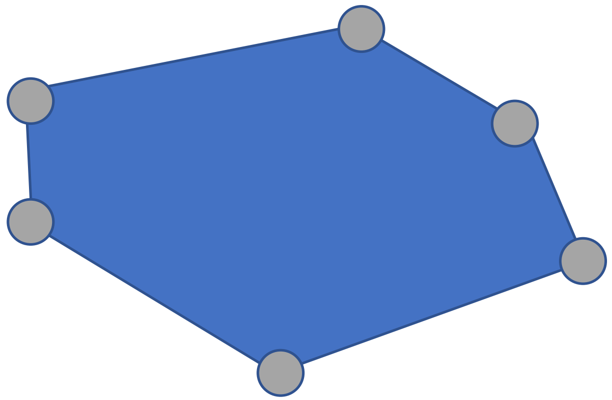
- Infeasible:



Co-funded by the
Erasmus+ Programme
of the European Union

Where is the optimal solution?

- It was proved it lies at vertex of the feasibility region
  - Region = convex polygon (2D), convex polyhedron (3D), convex polytope (n-D)
- Example in 2D:
  - 6 alternatives



- Objective function will help us choose the optimal point

Co-funded by the Erasmus+ Programme of the European Union

Using mathematics – The model

- Linear programming model is:
  - VARIABLES (usually $x_1, x_2 \dots x_n$)
  - CONDITIONS
    - Inequalities, equalities
  - OBJECTIVE FUNCTION
    - $F(x) \rightarrow MAX/MIN$
  - NONNEGATIVITY CONDITIONS
    - No variable can be lesser than 0

Co-funded by the
Erasmus+ Programme
of the European Union

Solution

Can be:

- Optimal
  - That‘s what we‘re looking for
  - Must be nonnegative and feasible
- Alternative
  - More optimal solutions = equal alternatives
- Sub-optimal
  - We can still find something better
- Infeasible
  - Out of feasible region – does not meet the conditions

Co-funded by the Erasmus+ Programme of the European Union

Tools

- Graphical solution
  - Only in 2D
  - 2 variables only
  - Unlimited number of conditions
- Numerical solution
  - Simplex method
    - Manual calculation in tables
    - Takes a little while
    - Only for smaller models
- Using computer
  - MS Excel (Solver, LinKOSA)
  - Pro software (Gurobi, Mathlab, Mathematica)

Co-funded by the Erasmus+ Programme of the European Union

DECISION METHODS

Poet and his wood

- he was allowed to buy (10 years ago) a cabin and 90 ha of woods
- he needs to walk the woods to keep his inspiration alive (muses do not always respond = sales from the woods can replenish empty wallet)
- he does not want to spend more than half of his time in the woods (the rest is for prose and sonnets)

Co-funded by the Erasmus+ Programme of the European Union

DECISION METHODS

- He has read about linear programming and desided to allocate scarce resources to optimize certain objectives
- Data
  - **40** ha of the land are covered with red-pine
  - **50** ha contain mixed hardwoods
  - since he bought these woods he has spent approx. **800** days managing the red-pine and **1500** days on the hardwoods
  - the total revenue **$36,000** from red-pine land and **$60,000** from the hardwoods

Co-funded by the Erasmus+ Programme of the European Union

DECISION METHODS

FRAME

Problem formulation

- the poet's objective is to maximize his revenues from the property (finite revenues = mean revenues per unit of time - year)

Max Z = $ of revenues per year

- Revenues (Z) arise from managing red-pine, or hardwoods, or both. Therefore, set of decision variables is:

X_1 = the number of hectares of red-pine to manage

X_2 = the number of hectares of hardwoods

- We seek the values of $\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ that make $\boldsymbol{Z}$ as large as possible

Co-funded by the Erasmus+ Programme of the European Union

DECISION METHODS

FRAME

Objective function

- the expresses the relationship between Z and the decision variables X_1 and X_2
- he has earned **$36,000** on **40** ha of red-pine and **$60,000** on **50** ha of hardwoods during the last 10 years (average earnings have been **90$/ha/y** for red-pine, **120$/ha/y** for hardwoods)

$$\text{Max } Z = 90\, X_1 + 120\, X_2$$

($/y) ($/ha/y) ($/ha/y)

Co-funded by the
Erasmus+ Programme
of the European Union

DECISION METHODS

FRAME

- Time constraints
  - expression of the constraint limiting this time no more than 180 days:

$$2 \quad X_1 \quad + \quad 3 \quad X_2 \leq 180$$

(d/ha/y) (ha) (d/ha/y) (ha) (d/y)

- Non negativity constraints
  - none of the decision variables may be negative, since they refer to areas

$$X_1 \geq 0 \quad \text{and} \quad X_2 \geq 0$$

Co-funded by the
Erasmus+ Programme
of the European Union

DECISION METHODS

- Final model
  - find the variables X_1 and X_2, which measure the number of hectares of red-pine and of hardwoods to manage, such that

$$\text{Max } Z = 90\,X_1 + 120\,X_2$$

subject to:

$X_1 \leq 40$

$X_2 \leq 50$

$2X_1 + 3X_2 \leq 180$

$X_1, X_2 \geq 0$